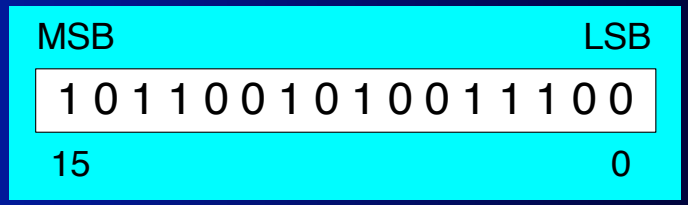
Data Representation

- Binary Numbers
  - Translating between binary and decimal
- Binary Addition
- Integer Storage Sizes
- Hexadecimal Integers
  - Translating between decimal and hexadecimal
  - Hexadecimal subtraction
- Signed Integers
  - Binary subtraction
- Character Storage

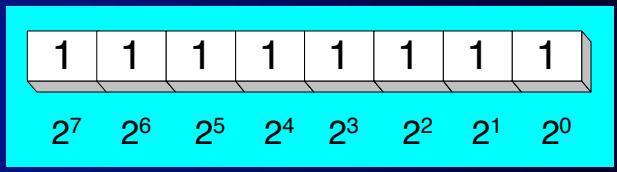
Binary Numbers

- Digits are 1 and 0
  - 1 = true
  - 0 = false
- MSB – most significant bit
- LSB – least significant bit
- Bit numbering:



Binary Numbers

- Each digit (bit) is either 1 or 0
- Each bit represents a power of 2:



Every binary number is a sum of powers of 2

Table 1-3 Binary Bit Position Values.

2^n	Decimal Value	2^n	Decimal Value
2^0	1	2^8	256
2^1	2	2^9	512
2^2	4	2^{10}	1024
2^3	8	2^{11}	2048
2^4	16	2^{12}	4096
2^5	32	2^{13}	8192
2^6	64	2^{14}	16384
2^7	128	2^{15}	32768

Translating Binary to Decimal

Weighted positional notation shows how to calculate the decimal value of each binary bit:

$$dec = (D_{n-1} \times 2^{n-1}) + (D_{n-2} \times 2^{n-2}) + \ldots + (D_1 \times 2^1) + (D_0 \times 2^0)$$

D = binary digit

binary 00001001 = decimal 9:

$$(1 \times 2^3) + (1 \times 2^0) = 9$$

Translating Unsigned Decimal to Binary

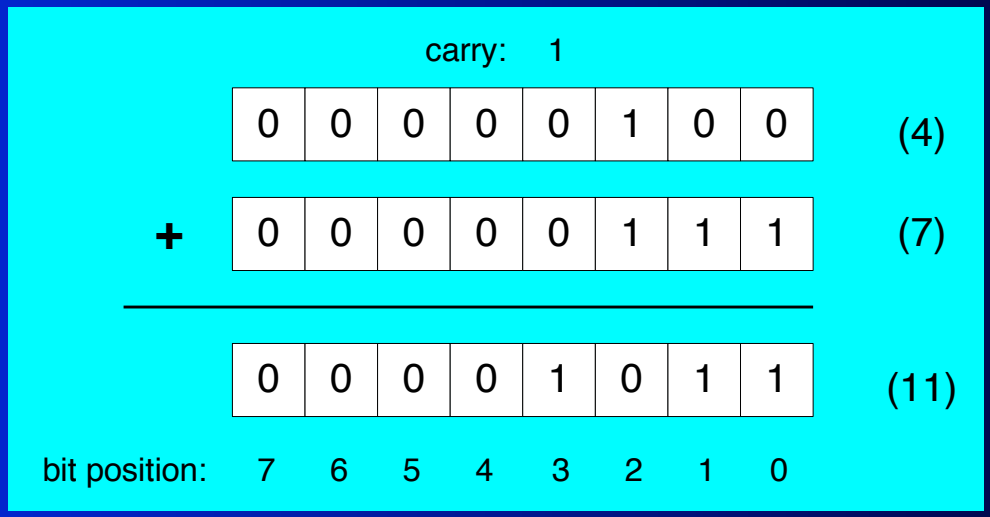
- Repeatedly divide the decimal integer by 2. Each remainder is a binary digit in the translated value:

Division	Quotient	Remainder
37 / 2	18	1
18 / 2	9	0
9 / 2	4	1
4 / 2	2	0
2 / 2	1	0
1 / 2	0	1

37 = 100101

Binary Addition

- Starting with the LSB, add each pair of digits, include the carry if present.



Integer Storage Sizes

Standard sizes:

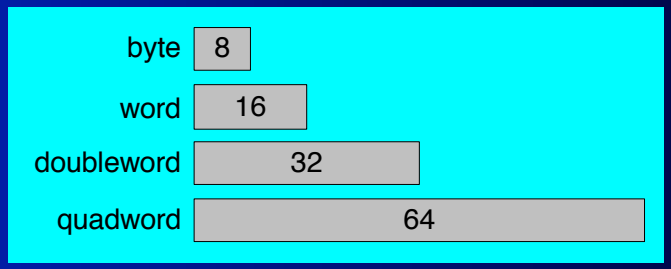


Table 1-4 Ranges of Unsigned Integers.

Storage Type	Range (low–high)	Powers of 2
Unsigned byte	0 to 255	0 to $(2^8 - 1)$
Unsigned word	0 to 65,535	0 to $(2^{16} - 1)$
Unsigned doubleword	0 to 4,294,967,295	0 to $(2^{32} - 1)$
Unsigned quadword	0 to 18,446,744,073,709,551,615	0 to $(2^{64} - 1)$

What is the largest unsigned integer that may be stored in 20 bits?

Hexadecimal Integers

Binary values are represented in hexadecimal.

Table 1-5 Binary, Decimal, and Hexadecimal Equivalents.

Binary	Decimal	Hexadecimal	Binary	Decimal	Hexadecimal
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	10	A
0011	3	3	1011	11	B
0100	4	4	1100	12	C
0101	5	5	1101	13	D
0110	6	6	1110	14	E
0111	7	7	1111	15	F

Translating Binary to Hexadecimal

- Each hexadecimal digit corresponds to 4 binary bits.
- Example: Translate the binary integer 000101101010011110010100 to hexadecimal:

1	6	A	7	9	4
0001	0110	1010	0111	1001	0100

Converting Hexadecimal to Decimal

- Multiply each digit by its corresponding power of 16:

  $dec = (D_3 \times 16^3) + (D_2 \times 16^2) + (D_1 \times 16^1) + (D_0 \times 16^0)$

- Hex 1234 equals $(1 \times 16^3) + (2 \times 16^2) + (3 \times 16^1) + (4 \times 16^0)$, or decimal 4,660.

- Hex 3BA4 equals $(3 \times 16^3) + (11 * 16^2) + (10 \times 16^1) + (4 \times 16^0)$, or decimal 15,268.

Powers of 16

Used when calculating hexadecimal values up to 8 digits long:

16^n	Decimal Value	16^n	Decimal Value
16^0	1	16^4	65,536
16^1	16	16^5	1,048,576
16^2	256	16^6	16,777,216
16^3	4096	16^7	268,435,456

Converting Decimal to Hexadecimal

Division	Quotient	Remainder
422 / 16	26	6
26 / 16	1	A
1 / 16	0	1

decimal 422 = 1A6 hexadecimal

Hexadecimal Addition

- Divide the sum of two digits by the number base (16). The quotient becomes the carry value, and the remainder is the sum digit.

		1	1
36	28	28	6A
42	45	58	4B
78	6D	80	B5

21 / 16 = 1, rem 5

Important skill: Programmers frequently add and subtract the addresses of variables and instructions.